Contextual Fundamentals, Models, and Active Management

Improving on one-size-fits-all.

Eric H. Sorensen, Ronald Hua, and Edward Qian

Eric H. Sorensen
is the president and chief executive officer of PanAgora Asset Management in Boston.
esorensen@panagora.com

Ronald Hua
is director of Dynamic Equity at PanAgora Asset Management.
rhua@panagora.com

Edward Qian
is a senior portfolio manager at PanAgora Asset Management.
eqian@panagora.com

Quantitative and fundamental money managers alike seek to find and construct portfolios of undervalued securities in the hope of delivering positive alpha in an efficient manner. Most understand that a given financial signal associated with specific stocks is often variably important. A variety of considerations may dictate how a specific factor should differentially influence the returns across stocks: 1) the nature of the mandate and investment guidelines, and 2) evidence that some factors work well (or poorly) depending on certain other characteristics of the stock.

An example of the first instance is the long-only constraint, which rules out return opportunities from short-selling. Such a restriction imposes a limit on how much return can be generated from undesirable stocks while leaving desirable stocks with their full return potential. An example of the second instance is the importance of earnings momentum for companies in mature businesses with relatively predictable growth rates versus companies in high potential growth and high-risk businesses.

We present quantitative methodologies that explicitly recognize that a quantitative factor like present value or the PE ratio is not "one size fits all." Starting with the investment objective function of maximization of information ratio (IR), we offer a modeling process that is more robust in linking signals with investment returns. The result is that factor weights vary across risk-specific universe subgroups.

CONTEXT

In practice, linking a stock's ranking signal or factor to expected return and assigning it an appropriate weight is a matter of *context*. The application of a timely security selection criterion is conditional. Simply—it depends. Many researchers demonstrate that value, as a selection variable, for example, is often conditional on type of firm, other non-value factors, investment horizon, or some other dimension. Beneish, Lee, and Tarpley [2001] and Sloan [2001] call this interdependence of security factors *contextual*.

Seasoned active managers know that value investing focuses on discovering cheap stocks with a balance of quality; growth investing often seeks to balance positive momentum with quality and cheapness. This anecdotal assertion finds substantiation in research. For example, Daniel and Titman [1999] find that momentum effects are stronger for growth stocks. Asness [1997] finds that value strategies work in general, but work less well for stocks with high momentum.

In a particularly relevant study, Scott, Stumpp, and Xu [1999] focus on prospect theory and investor overconfidence. They provide empirical evidence that rational value investors should emphasize cheapness (as in dogs), while growth investors should let winners run—with the prospect of future good news. Piotroski [2000] and Mohanram [2005] also show that we should focus on different sets of financial statement information when we analyze stocks with different book-to-price ratios.

Taken together, these studies (and others) point to how important it is to analyze the efficacy of alpha factors within carefully selected security universes—the contextual analysis of active strategies. We use published empirical results as a guide to study the relative importance of various alpha factors within different security contexts.

ANALYTICAL FRAMEWORK

To establish the importance of analyzing active strategies contextually, we derive a set of empirically estimated optimal weights of factor categories (cheapness, quality, and momentum) in securities partitioned along the dimensions of different risk characteristics (value, growth, or earnings variability). We derive optimal weights to achieve the highest information ratio (IR) in each of these contexts. Our hypothesis is that there can be significantly different optimal factor weights when conditioned on different risk characteristics.

The basic building block of our framework begins with the historical information coefficient (IC) of each factor. The IC is the periodic measure of the cross-sectional correlation between factor ranks (using normalized Z-scores) and subsequent return performance across a large sample of stocks. Effectively, ICs measure the average impact per stock on return associated with a portfolio that has a one-sigma exposure to a factor, other things equal.

We must emphasize that the IC we use is not the traditional *raw* IC, the correlation between the raw factor forecasts and subsequent returns. Instead, it is a *risk-adjusted* IC that strips out multiple systematic risk exposures that are not allowed in a portfolio mandate, and accommodates stock-specific risks. In Qian and Hua [2004] we show the *risk-adjusted* IC of a given factor provides a more accurate description of portfolio returns and is a more relevant measure of factor efficacy since the ratio of its average to its standard deviation equates approximately to the IR of an active strategy employing the factor.

We estimate the risk-adjusted IC by stripping out exposures to the market beta and market capitalization, two risk factors with high cross-sectional explanatory power, which the traditional equity mandate typically prohibits in generating alpha. To accommodate a modern *active strategy* incorporating many alpha sources (or alpha factors), in Sorensen et al. [2004] we extend the single-factor results in an analytical expression of the IR of a strategy with multiple alpha sources as well as the optimal factor weights with maximum IR.[1]

Sorensen et al. [2004] show that the optimal weights are a function of average ICs and IC covariances:

$$\boldsymbol{\omega} \propto \mathbf{V}^{-1}\,\overline{\mathbf{IC}} \qquad (1)$$

where $\boldsymbol{\omega}$ is the vector of factor weights; $\mathbf{V}^{-1}$ is the inverse of the covariance matrix of the ICs through time; and $\overline{\mathbf{IC}}$ is the vector of the averages of the risk-adjusted ICs.[2]

For practitioners, Equation (1) provides several insights regarding the relative importance of different factors. First, if one assumes the ICs are uncorrelated, the optimal weight of each factor is proportional to its ratio of the average of the ICs to the variance of the ICs, i.e., its own return-risk trade-off. Second, and more realistically, when correlations are non-zero, the relative importance of a factor depends not only on its average-to-variance ratio but also on the diversification benefit it provides. In other words, a factor should be deemed to be more important when 1) it provides a more accurate return forecast—a high average-to-variance ratio, and 2) it enhances the diversification benefit—low or even negative correlations with other factors.

We evaluate the interplay among different factor categories in an optimization framework. Our approach is to assess the relative importance of each category as it varies contextually across specific security contexts—partitions of a broad security universe along the dimensions of different risk characteristics. Our results should help practitioners construct a specific alpha model based on contextual research.

FACTOR CATEGORIES

John Burr Williams [1938] developed the first modern expression for the fundamental valuation of a stock's intrinsic value. His is a clear expression that a company's stock should achieve a market price that quantifies the present value of all potential future profitable operations of the firm that accrue to shareholders. Williams's work set the stage for more sophisticated treatments of risk and equilibrium theory, but simply taking the first derivative of his expression with respect to value determinants starts us on the way to the modern practice of security selection.

In its simplest form, valuation is a function of growth prospects, firm quality, and investor expectations. The calibration of firm quality and changes in expectations helps us qualify static measures of cheapness to move closer to intrinsic value. In our study we focus on three sets of variables: 1) cheapness (often referred to as valuation), 2) corporate quality, and 3) investor sentiment. These three each take a turn as the primary criterion in three separate basic investment strategies: *Value investors* subordinate all factors to price; *fundamental investors* subordinate other factors to corporate operations, honesty of financial reporting, and preservation of shareholder interests; *momentum investors* subordinate much to expectations reflected in recent price or earnings trends.

Exhibit 1 introduces five composite factors representing the three investing philosophies. To capture the essence of value investing, which buys cheap stocks, we create the *relative valuation* (RV) composite, encompassing two types of cheapness measures: the earnings yield, and the asset value. We call this factor relative valuation because cheapness is gauged in the context of a peer group; we use sector as the peer group for comparison.

To represent the premise of fundamental investing, we break the analysis of the enterprise profitability accruing to shareholders into three composite factors: 1) the *operating efficiency* (OE) factor measuring management's ability to generate shareholder value, 2) the *accounting accrual* (AA) factor measuring the accuracy and the honesty of a company's financial reporting practices, and 3) the *external financing* (EF) factor measuring the hazard that self-serving managers may pursue corporate expansions at the expense of shareholder wealth.

Finally, the philosophy of riding market sentiment in momentum investing is captured in the *momentum* factor (MO), which consists of measures of intermediate-term price momentum, earnings revision, and earnings surprise.[3]

EXHIBIT 1
Definition of Factor Composites

Composite	Factors
Valuation (RV)	book-to-price ratio sales-to-enterprise value earnings yield (historical) earnings yield (IBES FY1) EBIT-to-enterprise value
Operating Efficiency (OE)	increase in asset turnover ratio level of operating leverage cash flow from operation to sales
Accounting Accruals (AA)	accounting accruals (balance sheet) accounting accruals (cash flow statement)
External Financing (EF)	external financing-to-net operating assets debt issuance-to-net operating assets equity issuance-to-net operating assets share count increase
Momentum (MO)	six-month price momentum nine-month earnings revision earnings surprise score

SECURITY CONTEXTS

We illustrate the interplay among factors along three risk characteristic dimensions: value, growth, and earnings variability. Along each dimension, we select two non-overlapping security contexts with equal numbers of stocks—one includes securities with high loadings of that risk characteristic, and the other securities with low loadings. Hence, there are six security contexts: firms with high/low value measures, high/low growth rates, and high/low earnings variability.

We use the book-to-price ratio as our first risk dimension: value. The name *value* for the book-to-price ratio implies that it is associated with market inefficiency, but this is not relevant to contextual analysis. What is relevant is the interpretation provided by Fama and French [1996], who associate the book-to-price ratio with the investment quality or financial condition of a company. That is, we can interpret a low book-to-price ratio as an indication of high quality and a high book-to-price ratio as an indication of low quality.

High-quality companies are expected by investors to deliver superior return on investment (ROI), and their ex post ROI typically exceeds the average ROI of a broad universe. Low-quality companies usually face a difficult operating environment and are expected to deliver poorer operating results. Different competitive standing, superior versus inferior, often implies different challenges facing company management—one battles from a deteriorated competitive position to survive, while the other protects its competitive advantage by fending off competition. These intuitions are confirmed in research by Piotroski [2000] and Mohanram [2005]. Therefore, we argue that investors should also focus on a different set of factors when they evaluate the return appeal of companies with different book-to-price ratios.

Our second risk characteristic sorts companies on the basis of growth rate, creating contexts with high-growth and low-growth companies. The rationale behind this contextual dimension is well documented by Scott, Stumpp, and Xu [1999 and 2003]. Linking behavioral science findings with valuation theory, they show that momentum investing (riding winners and looking for good news) is more important in selecting high-growth stocks; low-growth stock selection should focus more on cheapness. The difference lies in how investors estimate the fair value of a business.

A fair value estimate typically comprises two parts: the present value of current business, and the present value of future growth opportunities. For a low-growth company whose future growth prospect is limited, the value of its current business dominates its fair value, and, more important, valuation ratios (i.e., cash-flow yield or earnings yield) provide an accurate ranking of the relative cheapness of its current business. For high-growth companies, the majority of the fair value comes from the present value of future growth opportunities. Thus factors that are capable of predicting the quality of future growth play a more prominent role in determining fair value.[4]

Combining this valuation reasoning with the observations that investors tend to underreact to news due to their overconfidence, Scott, Stumpp, and Xu [1999 and 2003] show that the earnings revision factor, a proxy for good news, is a consistent predictor of the excess returns of growth stocks.

Our last dimension differentiates companies along the earnings variability dimension. This contextual selection is inspired by the persistent predictability bias documented by Huberts and Fuller [1995]. They show that U.S. sell-side analysts tend to issue overly optimistic forecasts for companies whose earnings are harder to predict, but their forecasts are more realistic (if still optimistic) for companies with stable earnings in the past. Das, Levine, and Sivaramakrishnan [1998] provide a more rigorous examination of this phenomenon and derive the same conclusion. Beckers, Steliaros, and Thomson [2004] find the same bias in European analysts' forecasts.

In all, if earnings forecasts are less trustworthy for companies whose earnings are more variable, it is our conjecture that investors should look at the quality of earnings and the competence of management to remedy the deficiency of these forecasts. Similarly, investors should rely more on analysts' forecasts when they evaluate stable earnings companies because these forecasts are more reliable.

EMPIRICAL EXAMINATION OF CONTEXTUAL DYNAMICS

A series of empirical tests will illustrate the nature of contextual asset pricing. We use the Russell 1000 Index as the security universe for the period January 1997-September 2004. Data sources include 1) the Compustat quarterly database for financial characteristics; 2) the IBES QFS database for consensus earnings estimates; and 3) the Barra US E3 database for price, return, and risk factor characteristics.

Risk-Adjusted ICs

We first compare the risk-adjusted ICs between sample partitions according to the Barra definitions of value, growth, and earnings variability. Along these Barra risk dimensions, we compare the average and the variance of IC pertaining to high and low security contexts for each of the composite alpha factors in Exhibit 1.

Exhibit 2 presents these comparisons (15 in all—three risk dimensions and five alpha measures). We calculate two-sample t-tests for the mean difference and F-tests for the variance difference.

In Panel A, the return profile of the external financing factor (EF) is significantly different for high- and low-value stocks. Both two-sample t-test and F-test results are significant at the 1% level. For low-value (low book-to-price ratio) stocks, the IC is 0.015, compared to 0.044 for high-value stocks. This demonstrates that the way the external financing factor is priced is indeed context-dependent—more important for discounted firms than high-priced ones. (Note that discounted firm means high value, and high-priced firm means low value.)

EXHIBIT 2

Comparison of Risk-Adjusted ICs in Different Risk Dimensions

PANEL A: VALUE DIMENSION

	Mean		STD		Two Sample t Test		F Test			
	High	Low	High	Low	t	p value	F	pval	df(num)	df(denom)
RV	0.022	0.022	0.069	0.079	0.011	0.991	0.764	0.270	68	68
OE	0.032	0.040	0.047	0.037	-1.050	0.296	1.613	0.051	68	68
AA	0.027	0.042	0.043	0.050	-1.912	0.058	0.720	0.177	68	68
EF	0.044	0.015	0.041	0.057	3.460	0.001	0.504	0.005	68	68
MO	0.031	0.049	0.061	0.072	-1.577	0.117	0.711	0.163	68	68

PANEL B: GROWTH DIMENSION

	Mean		STD		Two Sample t Test		F Test			
	High	Low	High	Low	t	p value	F	pval	df(num)	df(denom)
RV	0.003	0.034	0.113	0.062	-2.046	0.043	3.318	0.000	68	68
OE	0.061	0.019	0.043	0.042	5.702	0.000	1.037	0.883	68	68
AA	0.044	0.022	0.060	0.039	2.461	0.015	2.450	0.000	68	68
EF	0.028	0.017	0.054	0.043	1.274	0.205	1.567	0.066	68	68
MO	0.059	0.023	0.092	0.072	2.571	0.011	1.623	0.048	68	68

PANEL C: VARIABILITY DIMENSION

	Mean		STD		Two Sample t Test		F Test			
	High	Low	High	Low	t	p value	F	pval	df(num)	df(denom)
RV	0.023	0.023	0.105	0.076	-0.025	0.980	1.911	0.008	68	68
OE	0.045	0.029	0.051	0.039	2.019	0.046	1.678	0.034	68	68
AA	0.033	0.032	0.049	0.036	0.151	0.880	1.848	0.012	68	68
EF	0.038	0.018	0.055	0.045	2.343	0.021	1.492	0.101	68	68
MO	0.034	0.038	0.094	0.074	-0.252	0.802	1.605	0.053	68	68

External financing costs and expected investment returns contribute to this contextual dependence. Dilution of shareholder wealth is most likely to occur when the invested firm is traded at a discount and starts pursuing capital increases through external financing, because the proceeds not only cost more to obtain but also generate lower returns for current shareholders.

Panel B shows that investors reward high-growth companies for conservative accounting (AA), high operating efficiencies (OE), and better return and earnings performance (MO). Cheapness of share price (RV) is an important return driver for low-growth companies; both the average and the standard deviation of ICs are significantly different from high-growth companies at the 5% level. Our empirical results are consistent with those documented by Scott, Stumpp, and Xu [1999]. We also highlight the importance of conservative accounting and operating efficiency as important return drivers for high-growth companies. Like Asness [1997], we find more than twice the average IC of momentum factor (MO) in the high-growth stocks than in the low-growth stocks.

Panel C focuses on the earnings variability dimension. Operating efficiency (OE) and external financing (EF) factors are more indicative of the future stock returns of companies with variable earnings, as shown in the two-sample t-tests, which are significant at the 5% level. The relative valuation (RV) and accounting accrual (AA) factors have almost identical average IC across the partitions, but their standard deviations of ICs, the risk endogenous to the active strategies of applying RV and AA, are significantly different.

Exhibit 2 overall is consistent with the theory that rational pricing is conditional. Univariate average IC comparisons over the 1987-2004 period indicate the market is more responsive to operating efficiency, conservative accounting, and positive earnings evidence in the case of high-growth or high-priced firms than in the case of low-growth ones. The market is much more focused on operating performance and shareholder-friendly managements when growth is at stake, and much less focused on cheapness of stock prices.

When we look at the differences in IC average and IC standard deviation across the three risk partitions, the growth dimension appears to induce the most contextual difference while the variability dimension induces the least.

IC Correlations

Exhibit 3 reports the IC correlation matrices for the five composite factors in each of six risk partitions. In each case, the numbers before the slash mark are correlations for higher partitions; the numbers after are for lower partitions.

Some general patterns are worth noting. First, the IC correlation between relative valuation (RV) and momentum (MO) is always negative, providing diversification benefit to an active strategy by including both factors. Second, the correlations among the three composite factors in the same quality category, i.e., operating efficiency (OE), accounting accruals (AA), and external financing (EF), are not only all positive in general, but also seem to be rather stable across the risk partitions. Third, the relative valuation factor tends to be little correlated and

EXHIBIT 3
Correlations of Risk-Adjusted ICs

PANEL A: VALUE DIMENSION

	OE	AA	EF	MO
RV	0.28 / 0.16	-0.22 / 0.21	-0.08 / 0.63	-0.11 / -0.44
OE		0.42 / 0.50	0.16 / 0.24	0.24 / 0.19
AA			0.21 / 0.09	0.17 / 0.14
EF				0.18 / -0.23

PANEL B: GROWTH DIMENSION

	OE	AA	EF	MO
RV	-0.22 / 0.19	0.14 / -0.08	0.45 / -0.08	-0.71 / -0.25
OE		0.36 / 0.25	0.16 / 0.27	0.28 / 0.21
AA			0.23 / 0.21	-0.18 / 0.01
EF				-0.32 / 0.26

PANEL C: VARIABILITY DIMENSION

	OE	AA	EF	MO
RV	-0.16 / 0.12	-0.18 / 0.19	0.19 / 0.29	-0.60 / -0.38
OE		0.30 / 0.37	0.26 / 0.38	0.48 / 0.10
AA			0.28 / 0.19	0.19 / 0.04
EF				0.05 / -0.23

Numbers before the slashmark show correlation for the high context, numbers after the slash show correlation for the low context.

often negatively with other factors.

In all, the market generally prices quality and momentum concurrently, while rotating between cheapness and momentum, each at the expense of the other, due to perhaps changes in risk aversion.

Panel A compares the two correlation matrices derived from the high- and low-value contexts. The correlations between RV and AA and between RV and EF show the biggest differences. In high-value stocks, the two correlations are –0.22 and –0.08, respectively; in low-value stocks, the two correlations are considerably higher, at 0.21 and 0.63, respectively.

The other notable difference is the correlation between momentum (MO) and external financing (EF). It is 0.18 in high-value stocks and –0.23 in low-value stocks.

Along the growth dimension (Panel B), again the relative valuation causes most of the correlation differences. Its correlations with OE, AA, and EF all flip signs across the partition. The correlation between RV and MO is negative in both partitions, but it is remarkably low at –0.71 among high-growth stocks.

Along the variability dimension (Panel C), the differences in correlation coefficients are smaller than those in Panels A and B. In aggregate, momentum (MO) is less correlated with other factors in low-variability stocks than in high-variability stocks while relative valuation (RV) is an exception.

Optimal Factor Weights and Their Differences

We solve for the optimal weights of the composite alpha factor using the IR maximization framework. We shall refer to a combination of alpha factors as an *alpha model.* In each of the six risk partitions, we find the optimal weights of the five composite factors using the IC averages and IC covariances over the whole sample period.

Given the differences of these inputs shown in Exhibit 2, we would naturally expect different alpha models in each high/low risk partition. But are these weight differences statistically significant?

We devise several ways to answer this question. First we perform several direct tests on the optimal weights themselves. Later, we test the performance differences induced by weighting differences, focusing on their alpha-producing capabilities.

To test the statistical significance of the difference between the optimal weights, we adopt a bootstrapping procedure similar to that introduced by Michaud [1998]. We resample with replacement the historical ICs jointly for all five composite alpha factors in each of the six security contexts; the sample size is the same as the number of time periods in the original sample.

In each sample, we then calculate average ICs and IC covariances of five factors along the different risk partitions, and derive IR-maximizing optimal weights. This is repeated 1,000 times to obtain 1,000 sets of optimal weights in each risk partition. By introducing sampling errors into the average ICs and the IC covariances, we translate the sampling errors of historical ICs into the sampling errors of model weighting. We deem a weight deviation significant if it is significantly higher than the sampling error.

The model weights can be compared individually for each factor, or jointly for all five factors together. For individual comparison, Exhibit 4 shows the average and the standard error of factor weights of 1,000 bootstrapping samples, again across the 15 samples—three risk factor partitions and five alpha factors. We also show the difference in optimal weights across the three risk dimensions in terms of aver-

EXHIBIT 4

Resampled Weight Comparison in Different Risk Dimensions

PANEL A: VALUE DIMENSION

	Mean		STD		Difference (High-Low)		
	High	Low	High	Low	Avg/Stdr	Avg	Stdr
RV	9.0	6.3	4.0	3.5	0.5	2.6	5.3
OE	16.7	46.4	6.0	8.9	-2.7	-29.7	10.8
AA	20.4	24.4	6.2	6.5	-0.4	-4.0	9.0
EF	43.0	5.1	7.9	4.8	4.1	37.9	9.3
MO	11.0	17.8	4.8	5.1	-1.0	-6.8	7.1

PANEL B: GROWTH DIMENSION

	Mean		STD		Difference (High-Low)		
	High	Low	High	Low	Avg/Stdr	Avg	Stdr
RV	3.7	22.8	2.4	7.3	-2.5	-19.1	7.6
OE	52.7	16.9	7.8	8.3	3.1	35.8	11.7
AA	16.7	33.3	5.0	8.8	-1.6	-16.6	10.1
EF	14.0	16.7	5.9	7.2	-0.3	-2.7	9.3
MO	12.9	10.3	4.0	5.0	0.4	2.6	6.3

PANEL C: VARIABILITY DIMENSION

	Mean		STD		Difference (High-Low)		
	High	Low	High	Low	Avg/Stdr	Avg	Stdr
RV	7.9	7.2	3.8	4.5	0.1	0.7	5.9
OE	36.1	27.0	7.4	6.5	0.9	9.1	10.0
AA	27.2	41.1	6.3	7.5	-1.4	-13.9	9.6
EF	22.5	10.5	6.6	5.1	1.4	12.0	8.4
MO	6.4	14.2	3.7	4.4	-1.4	-7.9	5.7

ages, standard errors, and their ratios. These ratios can be interpreted as a t-statistic; a value of above 2 or below –2 indicates statistical significance in mean differences.

While the results in Exhibit 4 are consistent with our interpretation of the univariate IC tests and correlation differences above, several observations are worth noting. First, model weights in the high-growth context (Panel B) and the low-value context (Panel A) are remarkably similar. Perhaps this points to a set of common challenges facing high-priced and high-growth firms, the most important being to maintain superior operating results captured by the operating efficiency factor (OE).

But we note the reverse inference does not apply—model weights in the high-value and the low-growth contexts are quite different. In the high-value context, the most prominent weight is in the external financing factor (43%), while in the low-growth context, the model weights are relatively similar for all five factors. Note the relative value is weighted 23% here, while it never receives more than 10% elsewhere.

Second, we notice that in the growth dimension (Panel B), while the relative value factor's weight is substantially higher in the low-growth than in the high-growth dimension, with a mean standard error ratio of –2.5, consistent with the results of Scott, Stumpp, and Xu [1999], the momentum factor's weight is only slightly higher in the high-growth half (12.9%) than in the lower half (10.3%). The reason for this is the higher strategy risk of the momentum factor (MO) in the high-growth context (Exhibit 2, Panel B) than in the low-growth context.

Third, Exhibit 4 unveils primary return drivers for each security context, should they exist. To facilitate the discussion, let's delineate primary drivers as factors that constitute more than 40% of a model. Contextual partition plays a significant role in governing the primary return driver, as it shifts from operating efficiency (OE) for both high-priced and high-growth firms, to conservative external financing (EF) for discounted firms, and to honest management, gauged by conservative earnings reporting practice (AA), for firms with a stable earnings stream. These contextual dynamics further highlight the inadequacy of the one-size-fits-all assumption of traditional quantitative models.

Fourth, across both value and growth dimensions, there are two factors with significant weights, operating efficiency (OE) and external financing (EF) in value and relative valuation (RV), and operating efficiency (OE) in growth. Across the variability dimension, though, none of the factors shows significant weight difference.

Finally, we note that the aggregated weight in the corporate quality category, i.e., the sum of weights in operating efficiency (OE), accounting accruals (AA), and external financing (EF), accounts for over 70% of the model weight in almost all cases. This confirms the importance of financial statement analysis in active equity management.

Model Distance

Exhibit 5 tests for significance in differences between the optimal weights jointly. For comparison, we first construct a static one-size-fits-all model without any contextual partition, using the same resampling procedure. The first row of Panel A shows the resampled efficient weights for this static model; the other rows show the weights from the last section.

To compare the factor weights jointly, we use two measures. The first measure is the *distance* between two models, defined as:

$$d = \sqrt{\frac{\Delta\omega \times \Delta\omega}{k}} \tag{2}$$

where $\boldsymbol{\Delta\omega} = \mathbf{w}_{\text{model 1}} - \mathbf{w}_{\text{model 2}}$ is the difference in model weights, and k equals five, the number of factors in the model. It is the root mean square of the optimal weight differences.

Panel B of Exhibit 5 displays the distances between different pairs of models. Several observations are worth making. First, the static model is most similar to the high-variability contextual model and most unlike the high-value contextual model. Second, comparing the two contextual models on the same risk dimension, the greatest model distance is for the value dimension, followed by the growth dimension; the variability dimension is subject to the smallest distance. Third, there is little distance between the high-growth model and the low-value model.

While the distance measure does not incorporate the sample error, the second measure does. Panels C and D of Exhibit 5 provide the chi-square statistics between models and their p-values. Note the statistics are not symmetric, as we are testing whether the mean of the resampled weights of one model belongs to the ensemble of the resampled weights of another model. When the models are interchanged, the ensemble is also changed, resulting in a different chi-square statistic. Appendix A provides for a detailed technical note.

Panel D reveals three interesting findings. First, as shown in the first row (first column), the static model is statistically different from the contextual models on the growth and the value dimensions at the 5% level.[5] The contextual models along the variability dimension are not statistically different from the static one.[6]

Second, a comparison of model weights of the high and low contexts for each risk dimension shows significant differences for value and growth dimensions, while the variability dimension is questionable. Third, further substantiating the observation that the high-growth model is similar to the low-value model, the p-value is either 0.28 when using the covariance from the low-value context or 0.26 when using the high-growth covariance—neither is significant.

CONTEXTUAL ALPHA MODEL— A PROMISING ALTERNATIVE APPROACH

We have confirmed the benefits of a contextual approach in building quantitative alpha models, and some of the results concerning the value and growth dimensions should be applicable to portfolio mandates with style benchmarks, because our partitions along these dimensions are partly consistent with how many style benchmarks are defined. But what about mandates with core benchmarks? Can we build a contextual model based on our analysis that beats the one-size-fits-all model?

We propose an approach that dynamically selects factor weightings conditioned on the risk characteristics. Then, we compare the performance of the contextual models constructed with this approach and the static model. As these models use the same set of factors, this comparison provides some insight into the added value of dynamic factor weightings.

We implement four variants of the contextual model: value, growth, variability, and comprehensive. The first three models are built with the single risk dimension (two security contexts) indicated by their names. For example, the growth contextual model derives its dynamic factor weightings from the high-growth and the low-growth contexts only. The factor weighting for a particular stock is a linear combination of the high-growth and low-growth model, and relative weights of the combination are determined by the stock's growth rate.

The *comprehensive* contextual model takes into account all three risk dimensions, thus generating return forecasts based on optimal weights from all six security contexts. Appendix B illustrates the contextual model implementation.

To provide a more efficient use of our limited data sample and to facilitate a fair performance comparison, we use a cross-validation procedure. That is, we first divide our sample periods into ten subperiods of equal length. We then designate one subperiod as the out-of-sample period, and the remaining nine subperiods become the in-sample period.

EXHIBIT 5
Pairwise Model Weight Comparison

PANEL A: MODEL WEIGHTS OF RESAMPLED EFFICIENT PORTFOLIOS

		RV	OE	AA	EF	MO
One-size	R1000	2.5	41.6	36.3	13.0	6.5
Value	High	9.0	16.7	20.4	43.0	11.0
	Low	6.3	46.4	24.4	5.1	17.8
Growth	High	3.7	52.7	16.7	14.0	12.9
	Low	22.8	16.9	33.3	16.7	10.3
Variability	High	7.9	36.1	27.2	22.5	6.4
	Low	7.2	27.0	41.1	10.5	14.2

PANEL B: MODEL DISTANCE

		One-size	Value		Growth		Variable	
		R1000	High	Low	High	Low	High	Low
One-size	R1000	0.0	21.2	9.4	11.7	12.7	7.1	8.7
Value	High	21.2	0.0	24.4	23.2	14.6	14.7	20.0
	Low	9.4	24.4	0.0	7.1	16.9	11.7	13.2
Growth	High	11.7	23.2	7.1	0.0	19.8	11.2	17.8
	Low	12.7	14.6	16.9	19.8	0.0	10.7	7.4
Variability	High	7.1	14.7	11.7	11.2	10.7	0.0	11.0
	Low	8.7	20.0	13.2	17.8	7.4	11.0	0.0

PANEL C: CHI-SQUARED STATISTICS

		One-size	Value		Growth		Variable	
		R1000	High	Low	High	Low	High	Low
One-size	R1000	0.0	31.8	13.2	19.8	13.5	5.6	7.7
Value	High	69.0	0.0	65.6	39.1	15.9	13.8	49.0
	Low	32.0	36.2	0.0	5.2	21.6	17.0	11.1
Growth	High	16.6	39.7	5.0	0.0	24.9	13.1	19.4
	Low	73.7	18.9	34.0	74.7	0.0	24.2	18.3
Variability	High	11.9	13.7	17.7	14.2	8.6	0.0	9.7
	Low	17.0	23.2	9.9	24.8	7.7	14.0	0.0

PANEL D: P-VALUE OF CHI-SQUARED TEST

		One-size	Value		Growth		Variable	
		R1000	High	Low	High	Low	High	Low
One-size	R1000	1.000	0.000	0.010	0.001	0.009	0.235	0.103
Value	High	0.000	1.000	0.000	0.000	0.003	0.008	0.000
	Low	0.000	0.000	1.000	0.264	0.000	0.002	0.026
Growth	High	0.002	0.000	0.282	1.000	0.000	0.011	0.001
	Low	0.000	0.001	0.000	0.000	1.000	0.000	0.001
Variability	High	0.018	0.008	0.001	0.007	0.072	1.000	0.045
	Low	0.002	0.000	0.041	0.000	0.102	0.007	1.000

EXHIBIT 6
Performance Comparison of Optimal Dollar-Neutral Portfolios

PANEL A: MODEL PERFORMANCE

	Static	Value	Growth	Variable	Comp.
Alpha	7.41%	8.53%	8.54%	7.95%	8.57%
IR	1.56	1.63	1.66	1.54	1.72

PANEL B: PAIRWISE PERFORMANCE COMPARISON

	Static	Value	Growth	Variable	Comp.
Static		-1.13% (**-4.39)	-1.13% (**-4.75)	-0.54% (**-3.64)	-1.16% (**-6.06)
Value	1.13% (**4.39)		0.00% (-0.02)	0.58% (*2.45)	-0.03% (-0.23)
Growth	1.13% (**4.75)	0.00% (0.02)		0.59% (**3.34)	-0.03% (-0.19)
Variability	0.54% (**3.64)	-0.58% (*-2.45)	-0.59% (**-3.34)		-0.62% (**-4.46)
Comp.	1.16% (**6.06)	0.03% (0.23)	0.03% (0.19)	0.62% (**4.46)	

**Significant at or below 0.05 level.*

***Significant at or below 0.01 level.*

While efficient model weights (for both the static and contextual models) are estimated in the in-sample period through our IR optimization framework, the scores (forecasts) are computed according to the estimated factor weights for the out-of-sample periods when model performance is compared. This exercise is repeated ten times for each of the ten subperiods. Out-of-sample results are then combined to calculate performance statistics.

While we realize this approach creates chronological inconsistency in terms of the sequencing of the in-sample, out-of-sample, periods, it is free of potential bias caused by a particular choice of in-sample, out-of-sample, periods.

Performance of Risk-Adjusted Portfolios

Exhibit 6 compares model efficacy in terms of the excess returns generated by dollar-neutral portfolios, a comparison that incorporates realistic portfolio optimization constraints. Rebalanced on a quarterly basis, portfolios are formed for each model aiming at the highest model score exposures, with annualized tracking error targeted at 5% and no exposure to market beta and size.

Panel A shows the excess return and information ratio of each model on an annual basis. While the static model has the lowest excess return and the comprehensive model the highest excess return and IR, all models generate excellent performance.

We also compare model performance in a pairwise manner with the average and the t-statistic of performance differences through time. Each cell in Panel B represents the excess performance of the *active* model indicated by the row title and the *benchmark* model indicated by the column title.

As shown in the first column, contextual modeling enhances portfolio returns over static modeling. The enhancement of quarterly returns ranges from 1.16% to 0.54%. According to the t-statistics (in parentheses), the comprehensive contextual model provides the most consistent outperformance with a t-statistic of 6.06, followed by the growth contextual model with a t-statistic of 4.75. Also worth noting is that incorporating either the value or the growth dimension captures a significant portion of performance improvement, as the comprehensive implementation outperforms either model by only 3 basis points annually, shown in the last row.

Finally, the superior ex post performance delivered by the value and growth models underscores the importance of the model distance test, which indicates significant difference from the static model for models along the value and the growth dimensions, but not the variability dimension. Perhaps the model distance test provides a way to select contextual models that are likely to deliver better ex post returns.

Asset Pricing Tests (Fama-MacBeth Regression)

Exhibit 7 documents the advantage of using contextual modeling from an asset pricing perspective. That is, incorporating contextual dependencies provides a better and more accurate description of the way stocks are priced. Following the commonly accepted analytical framework in asset pricing studies, we apply the Fama-MacBeth regression to estimated returns to model scores through time on a quarterly basis.

Panel A evaluates whether contextual models incorporate relevant asset pricing information that is not captured by the static score. In this test, the dependent

EXHIBIT 7
Fama-MacBeth Regression Tests

PANEL A: RESIDUAL CONTEXTUAL SCORES VERSUS STATIC SCORE

	beta	size	static	residual comp.	residual value	residual growth	residual variability
comprehensive	-0.262	-0.035	1.650	1.046			
	(-0.3)	(-0.1)	(12.9)	(6.8)			
value	-0.288	-0.069	1.649		0.937		
	(-0.3)	(-0.2)	(13.0)		(6.6)		
growth	-0.262	-0.018	1.653			0.970	
	(-0.3)	(-0.1)	(12.9)			(5.8)	
variability	-0.223	-0.023	1.661				0.773
	(-0.2)	(-0.1)	(13.0)				(4.5)

PANEL B: RESIDUAL STATIC SCORE VERSUS CONTEXTUAL SCORES

	beta	size	residual static	comp.	value	growth	variability
comprehensive	-0.263	-0.035	-0.559	1.915			
	(-0.3)	(-0.1)	(-3.8)	(14.2)			
value	-0.287	-0.068	-0.274		1.913		
	(-0.3)	(-0.2)	(-2.1)		(13.1)		
growth	-0.262	-0.018	-0.400			1.913	
	(-0.3)	(-0.1)	(-2.5)			(13.9)	
variability	-0.224	-0.023	-0.445				1.797
	(-0.2)	(-0.1)	(-2.7)				(12.9)

variable is three-month forward return, and the explanatory variables are beta, size, static model score, and the residual contextual score (the contextual score netting out the static score). The netting out allows for an orthogonal treatment, which distills the portion of asset pricing information included exclusively in the contextual score, thus providing a measure that isolates the incremental value added by the contextual modeling.[7]

The residual score of the comprehensive contextual model does indeed capture additional asset pricing information (t-statistic of 6.8). Similar results are found when the three risk dimension-specific models are tested; their t-statistics range from 6.6 to 4.5, all significant at the 1% level.

Panel B shows the result of a complementary question. Is the static model statistically dominated by contextual models in an asset pricing test? In other words, does the static score add value when orthogonalized by contextual scores?

To answer this question, we include the residual of static score and contextual scores in this set of Fama-MacBeth regressions. The residual score is computed by stripping the portion of variance of the static score that can be explained by the contextual score through an ordinary least squares regression, the same procedure used in the tests shown in Panel A.

Panel B shows the contextual score provides return forecasts that dominate the forecasts of the static model statistically. The return to the static score residual is not only negative but also statistically significant with a t-statistic of –3.8. Again, similar results are also found in tests of the three risk dimension-specific scores. The t-statistics in these three tests range from –2.1 to –2.7.

SUMMARY

A growing number of studies have documented empirical evidence supporting the notion that rational asset pricing is conditional. This raises a question as to the descriptive accuracy of the one-size-fits-all assumption behind the traditional quantitative alpha model, which perhaps shares the same origin as traditional multifactor risk models. To better capture cross-sectional pricing dynamics and improve the performance of active equity strategies, we propose an alternative approach to alpha modeling—contextual modeling.

The approach represents a three-step process: 1) selecting contextual dimensions that provide an adequate description of the conditional nature of how stocks are priced; 2) determining the optimal factor weightings for each security context; and 3) associating stocks with each security context to obtain final scores. While we have proven the merit of this approach, with our choice of contextual dimensions, alpha factors, IR-maximizing technique, and model construction method, this is by no means the only approach. Incorporating earnings variability as a contextual dimension, for example, seems to add marginal value to the static model.

We must also note that our contextual approach is essentially a piecewise linear model conditioned on risk dimensions. While this might be the simplest non-linear model and the least prone to data-mining, there may be other ways to construct non-linear forecasting models. Further research should provide a better understanding of this alternative alpha modeling framework.

APPENDIX A

Model Distance Test

To gauge the significance of weighting differences, the likelihood of *not* attributing the cause solely to chance, we bootstrap the information coefficient sample to simulate the inherent randomness of the weight estimation procedure by systematically introducing sampling errors into estimates. The bootstrapping procedure, similar to one introduced by Michaud [1998], samples historical ICs, with replacement, 1,000 times and derives 1,000 sets of optimal weights, one for each sample. This exercise is repeated for each security context to generate the set of resample weightings and the average of these weightings. This average, ω, is the efficient factor weights—a convention established by Michaud [1998].

To illustrate how model distance is determined and tested, let's assume that $\boldsymbol{\omega}_1$ and $\mathbf{W}_1$ are the vector of efficient factor weights and the ensemble of resampled model weightings for the first security context, respectively, and that $\boldsymbol{\omega}_2$ and $\mathbf{W}_2$ are those for the second context. The vector of weighting difference is simply the difference between $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_2$, $\Delta\boldsymbol{\omega} = \boldsymbol{\omega}_1 - \boldsymbol{\omega}_2$.

Equation (A) shows the chi-squared statistic when the weighting difference is tested against the sampling error generated from the second security context. The degree of freedom for this chi-squared test is the number of factors minus one, because factor weights sum to 100%:

$$\Delta\boldsymbol{\omega}' \times \Lambda^{-1} \times \Delta\boldsymbol{\omega} \tag{A}$$

where Λ^{-1} is the inverse of the covariance matrix for either $\mathbf{W}_1$ or $\mathbf{W}_2$.

Since a different covariance matrix, estimated from either $\mathbf{W}_1$ or $\mathbf{W}_2$, can be selected to compute the chi-squared statistic, significance test results may vary depending on the relative tightness of these covariances, although the same weighting difference is in question. Exhibit A shows a two-dimensional schematic plot of factor weights for a visual demonstration.

The weighting difference is significant when using the covariance of $\mathbf{W}_2$ whose distribution on the right is tighter, while the result is not significant with the more diffused distribution of $\mathbf{W}_1$. The dashed circles are the loci of significant distances for the two distributions.

APPENDIX B

Contextual Model Implementation

To implement contextual modeling, the mechanism determining how much influence each optimal model (derived from various security contexts) should impose on a particular security's return forecast ultimately governs dynamic weighting decisions on the security level. Proximity, the distance between a security and each security context, is our choice of a mechanism that generates a vector of model weights for each security.[8] Model weights are of inverse proportion to the proximity measure that defines the risk dimension—the closer the distance to a context, the greater weight is assigned to the corresponding model for that security. A security's contextual forecast is composed as a weighted average of scores, generated by the contextually derived models.

Contextual model design can consider one single risk dimension or accommodate many. An example will show a single dimension of growth for demonstration purposes.

Exhibit B-1 shows model weights for two hypothetical securities, A and B, using the proximity measure. Apparently, A is closer to the low-growth context while B is better described by the high-growth context. The contextual forecast of security A is a 90/10 blend of scores from high- and low-growth models, and it is 70/30 for security B.

Exhibit B-2 shows the derived factor weights for A and B, computed as the weighted average of the high-growth and the low-growth models. Factor weights of A resemble the low-growth model more, while B looks more like the high-growth one.

Simple extrapolation accommodates multiple risk dimensions, a comprehensive implementation. Exhibit B-3 provides a schematic description of the proximity measures for security S in a two-dimensional case, and the corresponding model weights are dependent on the distances from S to each context, d_1 through d_4. Exhibit B-4 shows both model weights associated with each dimension for the high and low contexts and contextual weights assigned to each risk dimension reflecting the relevance of such dimension in describing the return behavior of S.

While other constructs are possible, we choose equal weights for the two risk dimensions in this implementation. That is, the final contextual weights are simply an equally weighted average of the model weights in the two dimensions.

EXHIBIT A
Distribution of Factor Weights

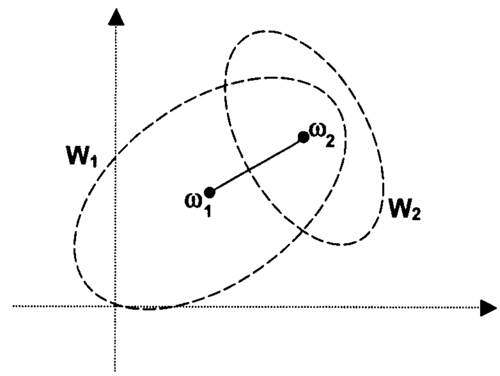

EXHIBIT B-1
Hypothetical Model Weights

	A	B
High Growth Model	10%	70%
Low Growth Model	90%	30%

EXHIBIT B-2
Derived Factor Weights

	A	B	High Growth	Low Growth
RV	20.8%	9.4%	3.6%	22.7%
OE	20.0%	41.8%	52.7%	16.3%
AA	32.1%	21.7%	16.6%	33.8%
EF	16.3%	14.9%	14.2%	16.5%
MO	10.9%	12.2%	12.9%	10.6%

EXHIBIT B-3
Multiple Risk Dimensions

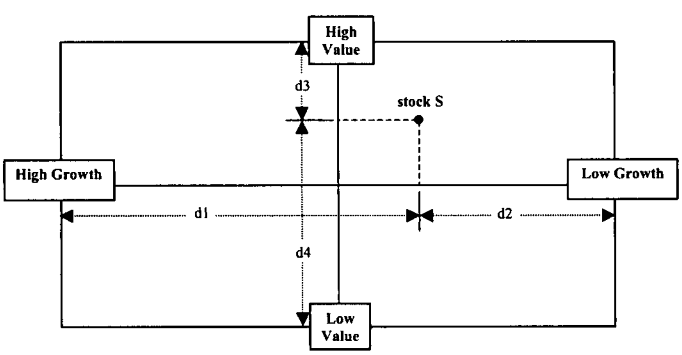

EXHIBIT B-4
Weights of Risk Dimensions

		Growth	Value
Model Weight	High	40%	90%
	Low	60%	10%
Contextual Weight		50%	50%

Careful readers will identify a number of important considerations that are omitted in this simple extrapolation. To name a few, the correlation of risk loadings, the significance of model distance, and the information ratio of each contextual model are some of the essential considerations constituting a more sophisticated implementation of an n-dimensional contextual model.

The Black-Litterman model [1990, 1992] provides a practical implementation that combines forecasts of multiple models via Bayesian statistics. It might be fruitful to apply that methodology to the contextual model using multiple risk partitions.

ENDNOTES

[1]They assume factor score correlations are stable over time.

[2]This result considers the benefits from diversification across a set of factors and reflects an underlying covariance structure of the returns to these factors. It shows optimal weights in proportional form. Algebraic multiplication can rescale these weights to sum to 100%, provided the initial sum is non-zero.

[3]These factors are relatively common; their calculations will be supplied on request.

[4]Equation (E-1) shows the fair value of current business, assuming that the current level of earnings can be sustained perpetually:

$$FV = \frac{E}{k} \tag{E-1}$$

where E is the normalized earnings expected to be sustained perpetually, and k is the discount rate, typically the cost of equity. The relative cheapness can be measured as the fair value divided by the market price, in Equation (E-2):

$$\frac{FV}{P} = \frac{1}{k} \cdot \frac{E}{P} \approx \frac{E}{P} \tag{E-2}$$

Assuming that the variance of the reciprocal of the discount rate is a lot smaller than the variance of the earnings yield cross-sectionally, the term $1/k$ can be treated as a constant and dropped from the equation. Now, the relative cheapness measure equals the ratio of earnings yield.

[5]Because the choice of inverse covariance is involved in presenting the numbers in Panels C and D, the off-diagonal terms of these two panels are not symmetric. The column title indicates the context whose covariance matrix is selected in the computation.

[6]The models along the variability dimension are different from the static model, viewed with resampled static model weights, but there are conflicting results tested from the perspective of the high-variability ensemble because the population of resampled weights for the high-variability stocks is more diffuse than that of the static model.

[7]The residual contextual score is derived as the residual term of a cross-sectional ordinary least squares regression where contextual score is the dependent variable, and static score is the only independent variable.

[8]Note that model weights for each security are different from factor weights for each contextually derived model. The

former conveys the relevance of each model in forecasting the future return of a particular security; the latter is calibrated on the basis of each factor's historical ICs pertaining to a particular security context.

REFERENCES

Asness, Clifford S. "The Interaction of Value and Momentum Strategies." *Financial Analysts Journal*, March/April 1997, pp. 29-36.

Beckers, Stan, Michael Steliaros, and Alexander Thomson. "Bias in European Analysts' Earnings Forecast." *Financial Analysts Journal*, March/April 2004, pp. 74-85.

Beneish, Messod D., Charles M.C. Lee, and Robin L. Tarpley. "Contextual Fundamental Analysis Through the Prediction of Extreme Returns." *Review of Accounting Studies*, Vol. 6 (2001), pp. 65-189.

Black, Fischer, and Robert Litterman. "Asset Allocation: Combining Investor Views with Market Equilibrium." Goldman, Sachs & Co., September 1990.

——. "Global Portfolio Optimization." *Financial Analysts Journal*, September/October 1992, pp. 28-43.

Daniel, Kent, and Sheridan Titman. "Market Efficiency in an Irrational World." *Financial Analysts Journal*, November/December 1999, pp. 28-40.

Das, Somnath, Carolyn B. Levine, and K. Sivaramakrishnan. "Earnings Predictability and Bias in Analysts' Earnings Forecasts." *The Accounting Review*, April 1998, pp. 277-294.

Fama, Eugene F., and Kenneth R. French. "Multifactor Explanations of Asset Pricing Anomalies." *Journal of Finance*, 51 (1996), pp. 55-84.

Huberts, Lex C., and Russell J. Fuller. "Predictability Bias in the U.S. Equity Market." *Financial Analysts Journal*, March/April 1995, pp. 12-28.

Michaud, Richard O. *Efficient Asset Management.* Cambridge: Harvard Business School Press, 1998.

Mohanram, Partha S. "Separating Winners from Losers among Low Book-to-Market Stocks using Financial Statement Analysis." *Review of Accounting Studies*, June-September 2005, p. 133.

Piotroski, Joseph D. "Value Investing: The Use of Historical Financial Statement Information to Separate Winners from Losers." *Journal of Accounting Research*, Vol. 38 (2000), pp. 1-41.

Qian, Edward, and Ronald Hua. "Active Risk and Information Ratio." *Journal of Investment Management*, Vol. 2 (2004), pp. 20-34.

Scott, James, Mark Stumpp, and Peter Xu. "Behavioral Bias, Valuation, and Active Management." *Financial Analysts Journal*, July/August 1999, pp. 49-57.

——. "News, Not Trading Volume, Builds Momentum." *Financial Analysts Journal*, March/April 2003, pp. 45-53.

Sloan, Richard G. "Discussion of: Contextual Fundamental Analysis Through the Prediction of Extreme Returns." *Review of Accounting Studies*, 6 (2001), pp. 1991-1995.

Sorensen, Eric H., Edward Qian, Ronald Hua, and Robert Schoen. "Multiple Alpha Sources and Active Management." *The Journal of Portfolio Management*, Winter 2004, pp. 39-45.

Williams, John Burr. The Theory of Investment Value." Cambridge: Harvard University Press, 1938.